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Perturbation expansion and initial-state correlations in non-equilibrium thermo-field dynamics

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Abstract. A perturbation scheme in non-equilibrium thermo-field dynamics is developed by taking into account the initial-state correlations. Two methods are proposed: in the first method the time dependence of amplitudes and their initial values are evaluated by two independent perturbation schemes while in the second method the two perturbation schemes are combined by use of a fictitious past.

1. Introduction

The path-ordered formalism has been widely used [1-4] as a perturbation scheme in non-equilibrium statistical physics. A recent review is given in [5] and recent applications to a relativistic theory are described in [6]. It is well known that the inclusion of non-Gaussian initial-state correlations is one of the main problems to be solved in order to obtain a consistent perturbation scheme [7, 8]. The initial correlations may be neglected if the initial state is given in the infinite past but they are important in the study of physical systems near the initial time.

When one wishes to include the initial correlation effects in a perturbation scheme in a consistent way, one needs to consider two kinds of perturbation, since the Hamiltonian and the initial density matrix are different.

In this paper, we propose a systematic perturbation method for non-equilibrium phenomena, including initial-state correlations. The method is based on thermo-field dynamics (TFD) [9] as recently reviewed in [10]. The advantage of TFD lies in the fact that this formalism is completely based on the operator formalism in a linear space and that the notion of states can be introduced. Recently, attempts to extend equilibrium TFD to non-equilibrium TFD have been proposed [11-14]. One of the present authors (HM) proposed a framework for specifying the initial-state vacuum with arbitrary particle distributions in the form of thermal-state conditions and developed a causal perturbation scheme [12]. In this paper we refine the treatment of the initial-state correlations. We propose two perturbation methods. One is based on a separate perturbative evaluation of the initial amplitudes and their time-development, which we will call the double expansion method. The other is based on a single perturbation scheme by considering a fictitious past for the evaluation of initial amplitudes, which we will call the unified expansion method.

The paper is organised as follows. In § 2, the framework of TFD in non-equilibrium phenomena is summarised. In § 3, a framework of perturbation schemes is presented in a convenient form for subsequent discussions. In § 4, the double expansion method

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is presented. In § 5, the unified expansion method is presented, in which the evaluation of initial values is unified as a time development in a fictious past. Section 6 is devoted to concluding remarks.

2. Non-equilibrium thermo-field dynamics

A quantum field system with a given particle distribution in space is analysed by the following framework in thermo-field dynamics (TFD) [9-14].

2.1. Quantum algebra

There are two sets of operators $\boldsymbol{a}$ and $\tilde{\boldsymbol{a}}$ which form independent algebras:

$$[A, \tilde{B}]_{\mp} = 0 \qquad A \in \boldsymbol{a} \text{ and } \tilde{B} \in \tilde{\boldsymbol{a}} \tag{2.1}$$

where the anticommutator $(+)$ is to be used if both A and $\tilde{B}$ are fermionic, otherwise the commutator $(-)$ must be taken. The algebra in $\boldsymbol{a}$ is the same as that required in the conventional quantum theory, so that Hermitian conjugation (denoted here by a dagger) is defined, the canonical relations are given and the existence of a Hamiltonian is assumed. Between $\boldsymbol{a}$ and $\tilde{\boldsymbol{a}}$ there exists a one-to-one mapping, called the tilde conjugation, which consists of the following rules:

$$\text{(i)} \quad [A_1 A_2]^{\sim} = \tilde{A}_1 \tilde{A}_2 \tag{2.2a}$$

$$\text{(ii)} \quad [C_1 A_1 + C_2 A_2]^{\sim} = C_1^* \tilde{A}_1 + C_2^* \tilde{A}_2 \tag{2.2b}$$

$$\text{(iii)} \quad [A^{\dagger}]^{\sim} = \tilde{A}^{\dagger} \tag{2.2c}$$

$$\text{(iv)} \quad [A]^{\sim} = A. \tag{2.2d}$$

2.2. Thermal state condition

There exists a vacuum $|G\rangle$. Physical amplitudes are given by vacuum expectation values of operators in $\boldsymbol{a}$. The vacuum $|G\rangle$ is specified at the initial time t_0 by a Hermitian operator $\Omega(t_0)$ in $\boldsymbol{a}$ as

$$A(t_0)|G\rangle = \sigma \exp(\hat{\Omega}(t_0))\tilde{A}^{\dagger}(t_0)|G\rangle \tag{2.3}$$

where

$$\hat{\Omega}(t_0) = \Omega(t_0) - \tilde{\Omega}(t_0). \tag{2.4}$$

$\Omega(t_0)$ is expressed in terms of operators at t_0, $A(t_0)$ is an arbitrary operator at t_0 and $\sigma = 1$ for bosonic A and $\sigma = \mathrm{i}$ for fermionic A. The thermal-state condition (2.3) leads to

$$A(t)|G\rangle = \sigma \exp(\hat{\Omega}(t_0))\tilde{A}^{\dagger}(t)|G\rangle \tag{2.5}$$

where

$$A(t) = \exp(\mathrm{i}H(t - t_0))A(t_0)\exp(-\mathrm{i}H(t - t_0)). \tag{2.6}$$

The operator $\Omega(t_0)$ fixes the particle distribution at the initial time t_0. It can be shown that the above framework leads to the relation [12]

$$\langle G|A(t)\ldots B(t')|G\rangle = \frac{\mathrm{Tr}[\exp(-2\Omega(t_0))A(t)\ldots B(t')]}{\mathrm{Tr}[\exp(-2\Omega(t_0))]}. \tag{2.7}$$

The thermal doublet $A^{\alpha}(t)$ is defined as

$$A^{\alpha}(t)=\begin{cases} A(t) & \alpha=1 \\ \tilde{A}^{+}(t) & \alpha=2. \end{cases} \tag{2.8}$$

The time development of $A^{\alpha}(t)$ is obtained from the rule (2.2) and is summarised as

$$A^{\alpha}(t)=\exp(\mathrm{i}\hat{H}(t-t_0))A^{\alpha}(t_0)\exp(-\mathrm{i}\hat{H}(t-t_0)) \tag{2.9}$$

where

$$\hat{H}=H-\tilde{H}. \tag{2.10}$$

We note that, when a time-dependent external disturbance exists, the Hamiltonian can depend on time through coefficients. In this case the Hamiltonian is expressed by operators at t_0 as $H=H(A(t_0);\,t)$ and will be denoted by $H_t(t_0)$ (or in short notation H_t). Naturally, (2.9) is modified as

$$A^{\alpha}(t)=\hat{U}(t,\,t_0)^{-1}A^{\alpha}(t_0)\hat{U}(t,\,t_0) \tag{2.11}$$

with

$$\hat{U}(t,\,t_0)=T\exp\left(-\mathrm{i}\int_{t_0}^{t}\mathrm{d}t'\,\hat{H}_{t'}(t_0)\right). \tag{2.12}$$

3. Generalised Gell-Mann–Low formula

In this section we will give a perturbation scheme in TFD in a convenient form for the subsequent discussions. In non-equilibrium TFD, we deal with two operators H_t and $\Omega(t_0)$, which contain, in general, interaction parts. For example, if the initial state is prepared as equilibrium with an applied external force which is switched off at t_0, then $\Omega(t_0)$ may be expressed as $\Omega(t_0)=\frac{1}{2}\beta(H(t_0)+F(t_0))$ with $F(t_0)$ representing an external effect. Therefore, we must develop a perturbation scheme to calculate time development together with thermal-state conditions.

Let us assume that H_t and Ω are separated into an unperturbed part and an interaction part

$$H_t=H_{0t}+H_{1t} \tag{3.1}$$

$$\Omega=\Omega_0+\Omega_1. \tag{3.2}$$

Hereafter, we omit, for simplicity, writing the explicit dependence on t_0, unless it is necessary. Note that, even if H_t is time independent, H_{0t} and H_{1t} can be time dependent. The unperturbed Ω_0 defines the unperturbed initial state $|G_0\rangle$ which satisfies the thermal-state condition

$$A(t_0)|G_0\rangle=\sigma\exp(\hat{\Omega}_0)\tilde{A}^{+}(t_0)|G_0\rangle \tag{3.3}$$

where $A(t_0)$ is an arbitrary operator at t_0.

Let us introduce a combined Hamiltonian $\mathscr{H}_t$ defined by

$$\mathscr{H}_t=H_t\theta(t-t_0)+H_{\Omega}\theta(t_0-t) \tag{3.4}$$

where θ is the step function and H_{Ω} is defined by

$$H_{\Omega}=2\beta^{-1}\Omega \tag{3.5}$$

with β being a parameter related to the inverse of temperature. Then we define an operator whose time dependence is determined by $\hat{\mathscr{H}}_t$

$$\psi_H^\alpha(t) = \hat{u}(t, t_0)^{-1}\psi_H^\alpha(t_0)\hat{u}(t, t_0) \tag{3.6}$$

where

$$\hat{u}(t, t_0) = T\exp\left(-\mathrm{i}\int_{t_0}^{t} \mathrm{d}t'\,\hat{\mathscr{H}}_{t'}\right) \tag{3.7}$$

and

$$\hat{\mathscr{H}}_t = \mathscr{H}_t - \tilde{\mathscr{H}}_t. \tag{3.8}$$

For $t \geqslant t_0$, $\psi_H^\alpha(t)$ represents the Heisenberg field. Corresponding to (3.1) and (3.2), we have

$$\hat{\mathscr{H}}_t = \hat{\mathscr{H}}_{0t} + \hat{\mathscr{H}}_{1t} \tag{3.9}$$

with

$$\hat{\mathscr{H}}_{0t} = \theta(t - t_0)\hat{H}_{0t} + \theta(t_0 - t)\hat{H}_{\Omega 0} \tag{3.10}$$

and

$$\hat{H}_{1t} = \theta(t - t_0)\hat{H}_{1t} + \theta(t_0 - t)\hat{H}_{\Omega 1}. \tag{3.11}$$

Recall the formula

$$\hat{u}(t, t_0) = \hat{u}_0(t, t_0)\hat{u}(t, t_0) \tag{3.12}$$

where

$$\hat{u}_0(t, t_0) = T\exp\left(-\mathrm{i}\int_{t_0}^{t} \mathrm{d}t'\,\hat{\mathscr{H}}_{0t'}\right) \tag{3.13}$$

and

$$\hat{u}(t, t_0) = T\exp\left(-\mathrm{i}\int_{t_0}^{t} \mathrm{d}t'\,\hat{\mathscr{H}}_1(t')\right) \tag{3.14}$$

with

$$\hat{\mathscr{H}}_1(t) = \hat{u}_0(t, t_0)^{-1}\hat{\mathscr{H}}_{1t}\hat{u}_0(t, t_0). \tag{3.15}$$

By noting that operators defined by

$$[A^\alpha]_G = \exp(\tilde{\Omega})A^\alpha \exp(-\tilde{\Omega}) \tag{3.16}$$

satisfy the thermal-state condition

$$\langle G|[A]_G = \sigma^*\langle G|[\tilde{A}^+]_G \tag{3.17}$$

we have

$$\langle G|[TA_H^\alpha(t)\ldots B_H^\gamma(t')]_G|G\rangle$$
$$= \langle G|[T\hat{u}(t_B, t_0)A^\alpha(t)\ldots B^\gamma(t')]_G|G\rangle \tag{3.18}$$

where $A^\alpha(t)$ is a field in the interaction representation defined by

$$A^\alpha(t) = \hat{u}_0(t, t_0)^{-1}A_H^\alpha(t_0)\hat{u}_0(t, t_0) \tag{3.19}$$

and t_B is a time larger than $(t\ldots t', t_0)$.

Next we rewrite the vacuum $|G\rangle$ in terms of the vacuum of the interaction representation $|G_0\rangle$, following the same procedure presented in [10, 15]. By use of the thermal-state condition (3.3), we can show that the state $\exp(-\tilde{\Omega})\exp(\tilde{\Omega}_0)|G_0\rangle$ satisfies the same thermal-state condition as $|G\rangle$. Therefore we have

$$|G\rangle = \exp(-\tilde{\Omega}(t_0))\exp(\tilde{\Omega}_0(t_0))|G_0\rangle C(\Omega) \tag{3.20}$$

with $C(\Omega)$ being a renormalisation constant. Since

$$\exp(-2\tilde{\Omega}(t_0))\exp(2\tilde{\Omega}_0(t_0)) = \tilde{u}(t_0, t_0 - i\beta) \tag{3.21}$$

with

$$\tilde{u}(t_0, t_0 - i\beta) = T_c \exp\left(i\int_{t_0-i\beta}^{t_0} dz\, \tilde{H}_{\Omega I}(z)\right) \tag{3.22}$$

$$\tilde{H}_{\Omega I}(z) = \exp(-i\tilde{H}_{\Omega 0}(z-t_0))\tilde{H}_{\Omega I}(t_0)\exp(i\tilde{H}_{\Omega 0}(z-t_0)) \tag{3.23}$$

and T_c being the path-ordered product, and since, for $t_A \leqslant t_0$,

$$[\hat{u}(t_A, t_0)\tilde{u}(t_0, t_0 - i\beta)]_{G0}|G_0\rangle = [u(t_A, t_A + i\beta)]_{G0}|G_0\rangle \tag{3.24}$$

then (3.18) is rewritten as

$$\langle G|[TA_H^\alpha(t)\ldots B_H^\gamma(t')]_G|G\rangle = \frac{\langle G_0|[Tu(t_B, t_A)A^\alpha(t)\ldots B^\gamma(t')u(t_A, t_A + i\beta)]_{G0}|G_0\rangle}{\langle G_0|[Tu(t_B, t_A)u(t_A, t_A + i\beta)]_{G0}|G_0\rangle}. \tag{3.25}$$

The renormalisation constant $C(\Omega)$ was determined to satisfy $\langle G|G\rangle = 1$. Note that, in (3.25), the final time t_B is larger than $t, \ldots, t', t_0$ and the initial time t_A is smaller than $t, \ldots, t', t_0$. The formula (3.25) is the generalised Gell-Mann–Low formula which will be used in the following discussions.

4. Double expansion method

Let us take in the formula (3.25)

$$t_B \to \infty \qquad t_A = t_0 \qquad t, \ldots, t' > t_0. \tag{4.1}$$

Then for $t > t_0$, the Hamiltonian is H_t and for the operator $u(t_0, t_0 + i\beta)$, the Hamiltonian is H_Ω.

Let us assume that Ω_0 is expressed by operators, (ψ_i^+, ψ_i), in a diagonal form

$$\Omega_0 = -\sum_i \tfrac{1}{2}\psi_i^+(\ln f_i)\psi_i \tag{4.2}$$

where f_i is a c number. Assuming that ψ_i and ψ_i^+ satisfy canonical relations, we have

$$\psi_i|G_0\rangle = \sigma_i\sqrt{f_i}\tilde{\psi}_i^+|G_0\rangle. \tag{4.3}$$

The operators defined by

$$a_i = (1 - \sigma_i^2 f_i)^{-1/2}(\psi_i - \sigma_i\sqrt{f_i}\tilde{\psi}_i^+) \tag{4.4a}$$

$$\tilde{a}_i = (1 - \sigma_i^2 f_i)^{-1/2}(\tilde{\psi}_i - \sigma_i^*\sqrt{f_i}\psi_i^+) \tag{4.4b}$$

annihilate the vacuum $|G_0\rangle$

$$a_i|G_0\rangle = \tilde{a}_i|G_0\rangle = 0 \tag{4.5}$$

and satisfy the same canonical relations as ψ_i. Therefore, we can construct a complete set of state vectors on $|G_0\rangle$ by a cyclic operation of $a_i^\dagger$ and $\tilde{a}_i^\dagger$. Then we have

$$1 = \sum_{n,m} \phi_{nm}^\dagger |G_0\rangle\langle G_0| \phi_{nm} \tag{4.6}$$

where ϕ_{nm} is the abbreviation of $\phi_n^\dagger \tilde{\phi}_m^\dagger$ with $\phi_n^\dagger$ being a complete set of operators constructed from $\{a_i^\dagger\}$ and $\tilde{\phi}_m^\dagger$ from $\{\tilde{a}_i'\}$. The completeness relation (4.6) in the interaction representation can be also expressed as

$$1 = \sum_{n,m} (\varphi_{nm}^*)_{G0} |G_0\rangle\langle| (\varphi_{nm})_{G0} \tag{4.7}$$

with

$$\varphi_{nm} = \exp(-\tilde{\Omega}_0)\phi_{nm} \exp(\tilde{\Omega}_0) \tag{4.8a}$$

$$\varphi_{nm}^* = \exp(-\tilde{\Omega}_0)\phi_{nm}^\dagger \exp(\tilde{\Omega}_0). \tag{4.8b}$$

The operator φ_{nm} is constructed from the operators defined by

$$\alpha_1 = \exp(-\tilde{\Omega}_0) a_i \exp(\tilde{\Omega}_0) \tag{4.9a}$$

$$\tilde{\alpha}_i = \exp(-\tilde{\Omega}_0)\tilde{a}_i \exp(\tilde{\Omega}_0) \tag{4.9b}$$

and φ_{nm}^* is constructed from

$$\alpha_i^* = \exp(-\tilde{\Omega}_0) a_i^\dagger \exp(\tilde{\Omega}_0) \tag{4.10a}$$

$$\tilde{\alpha}_i^* = \exp(-\tilde{\Omega}_0)\tilde{\alpha}_i^\dagger \exp(\tilde{\Omega}_0). \tag{4.10b}$$

The α operators are related to the original ψ operators by the relations

$$\begin{pmatrix} \alpha_i \\ \tilde{\alpha}_i^* \end{pmatrix} = V_{\sigma i}(f_i) \begin{pmatrix} \psi_i \\ \tilde{\psi}_i^\dagger \end{pmatrix} \qquad (\alpha_i^*, \tilde{\alpha}_i) = (\psi_i^\dagger, \tilde{\psi}_i) V_{\sigma i}(f_i)^* \tag{4.11}$$

with

$$V_{\sigma i}(f_i) = (1 - \sigma_i^2 f_i)^{-1/2} \begin{pmatrix} 1 & \sigma_i f_i \\ \sigma_i & 1 \end{pmatrix}. \tag{4.12}$$

By inserting (4.7) in the formula (3.25) with (4.1), we have

$$\begin{aligned} &\langle G|[TA_H^\alpha(t) \dots B_H^\gamma(t')]_G|G\rangle \\ &\quad = \sum \langle G_0|[T\hat{u}(\infty, t_0) A^\alpha(t) \dots B^\gamma(t') \varphi_{nm}^*(t_0)]_{G0}|G_0\rangle \\ &\qquad \times \frac{\langle G_0|[\varphi_{nm}(t_0) u(t_0, t_0 + i\beta)]_{G0}|G_0\rangle}{\langle G_0|[u(t_0, t_0 + i\beta)]_{G0}|G_0\rangle} \end{aligned} \tag{4.13}$$

where we have used

$$\langle G_0|[T\hat{u}(\infty, t_0) u(t_0, t_0 + i\beta)]_{G0}|G_0\rangle = \langle G_0|[u(t_0, t_0 + i\beta)]_{G0}|G_0\rangle. \tag{4.14}$$

Since φ_{nm} and φ_{nm}^* are expressed by the original fields, ψ_i, $\psi_i^\dagger$, ψ_i, $\psi_i^\dagger$, at t_0, through (4.11), the calculation of (4.13) is divided into two steps. At first we calculate perturbatively

$$\langle G_0|[T\hat{u}(\infty, t_0) A^\alpha(t) \dots B^\gamma(t') C^\delta(t_0)]_{G0}|G_0\rangle$$

and

$$\langle G_0|[C^\delta(t_0) u(t_0, t_0 + i\beta)]_{G0}|G_0\rangle$$

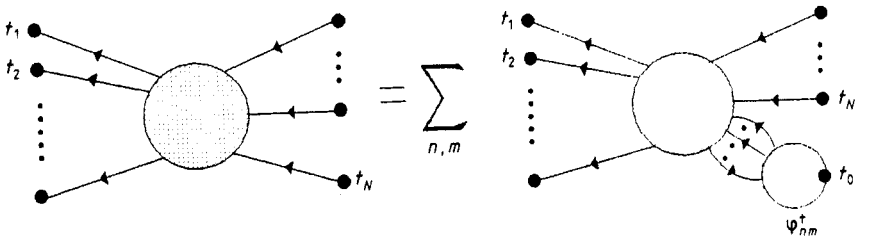

Figure 1. Initial-state correlations.

according to separate perturbation schemes and then we superpose them with proper weights. As shown in figure 1, the N-point function is expressed as the superposition of $(N+l)$-point functions with l values of time fixed at $t = t_0$.

When the initial time is chosen in the infinite past ($t_0 \to -\infty$) or in a sufficient past compared to the characteristic relaxation time, $A^{\alpha}(t) \ldots B^{\gamma}(t')$ and $\varphi^{*}_{nm}(t_0)$ are disconnected except for $\varphi_0 = 1$ due to the decaying property

$$\frac{\langle G_0|[T\hat{u}(\infty, t_0)A^{\alpha}(t) \ldots B^{\gamma}(t')u(t_0, t_0+i\beta)]_{G0}|G_0\rangle}{\langle G_0|u(t_0, t_0+i\beta)|G_0\rangle} \underset{t_0 \to -\infty}{\longrightarrow} \langle G_0|[T\hat{u}(\infty, -\infty)A^{\alpha}(t) \ldots B^{\gamma}(t')]_{G0}|G_0\rangle \tag{4.15}$$

that is, the initial correlation is negligible. This corresponds to the perturbation scheme of Keldysh in the path-ordered formalism [2, 16]. It is worthy of note that, in this case, only the first term in the right-hand side of (3.10) survives. Equation (4.15) is very similar to the corresponding expression for systems in thermal equilibrium, the only difference being the possible time dependence of the coefficients in H_{1I} and H_{0I}.

5. A unified perturbation method

The formula (4.13) is convenient in the sense that independent perturbation schemes are usable for the valuation of time dependence and initial conditions, although one must sum up contributions coming from the initial-state conditions. Therefore when such contributions are small, this formula is useful, for example, in the case when the time variables involved are sufficiently far from the initial time. However, sometimes one may need the infinite sum of corrections. In this case it is more convenient to develop a perturbation scheme in a unified way.

In formula (3.24) take $t_B \to \infty$ and $t_A \to -\infty$. We have

$$\langle G|[TA^{\alpha}_{H}(t) \ldots B^{\gamma}_{H}(t')]_{G}|G\rangle = \frac{\langle G_0|[T\hat{u}(\infty, -\infty)A^{\alpha}(t) \ldots B^{\gamma}(t')u(-\infty, -\infty+i\beta)]_{G0}|G_0\rangle}{\langle G_0|u(-\infty, -\infty+i\beta)|G_0\rangle}. \tag{5.1}$$

From (3.9) and (3.10), the unperturbed Hamiltonian $\hat{\mathcal{H}}_{0I}$ and the interaction Hamiltonian $\hat{\mathcal{H}}_{1I}$ change at $t = t_0$.

Let us assume that $H_{\Omega 0}$ and H_{0I} are given as bilinear functions of canonical variables $(\psi_i, \psi^{\dagger}_i)$ $([\psi_i, \psi^{\dagger}_j]_{-\sigma^2} = \delta_{ij})$ as

$$H_{\Omega 0} = \sum \omega_i \psi^{\dagger}_i \psi_i \tag{5.2}$$

$$H_{0I} = \sum \psi^{\dagger}_i \varepsilon_{ij} \psi_j. \tag{5.3}$$

Then f_i in (4.2) is given by

$$f_i = \exp(-\beta\omega_i). \tag{5.4}$$

By use of the matrix function $U(t, t_0)$ which satisfies

$$i\frac{\partial}{\partial t}U_{ij}(t, t_0) = \begin{cases} \varepsilon_{ik}(t)U_{kj}(t, t_0) & \text{for } t > t_0 \quad (5.5a) \\ \omega_i U_{ij}(t, t_0) & \text{for } t < t_0 \quad (5.5b) \end{cases}$$

and

$$U_{ij}(t_0, t_0) = \delta_{ij} \tag{5.6}$$

we can write

$$\psi_i^{\alpha}(t) = U_{ik}(t, t_0)\psi_k^{\alpha}(t_0) \qquad \psi_i^{\dagger\alpha}(t) = \psi_k^{\dagger\alpha}(t_0)U_{ki}(t_0, t). \tag{5.7}$$

Then, the unperturbed propagator is given as

$$\begin{aligned} &\langle G_0|[T\psi_i^{\alpha}(t)\psi_j^{\dagger\alpha}(t')]_{G0}|G_0\rangle \\ &\quad = U_{ik}(t, t_0)V_{\sigma}^{\alpha\alpha'}(f_k)[\tfrac{1}{2}(1+\tau)\theta(t-t') + \tfrac{1}{2}\sigma^2(1-\tau)\theta(t'-t)]^{\alpha'\gamma'} \\ &\qquad \times V_{\sigma}^{\gamma'\gamma *}(f_k)U_{kj}(t_0, t') \qquad (5.8a) \\ &\quad = U_{ik}(t, t_0)\left[\theta(t-t')\frac{1}{1-\sigma^2 f_k}\begin{pmatrix} 1 & \sigma^* f_k \\ \sigma & 1 \end{pmatrix}\right. \\ &\qquad \left. + \sigma^2\theta(t'-t)\frac{1}{1-\sigma^2 f_k}\begin{pmatrix} f_k & \sigma f_k \\ \sigma^* & 1 \end{pmatrix}\right]U_{kj}(t_0 t') \qquad (5.8b) \end{aligned}$$

where $V_{\sigma}(f)$ is given by (4.12). The change of the Hamiltonian at $t = t_0$ inevitably makes the phenomena time dependent, even if the external perturbations are time independent. Therefore vacuum expectation values are not functions of relative times only.

As in the equilibrium case [15], the end operator $u(-\infty, -\infty + i\beta)$ in (5.1), is disconnected from other operators due to damping properties for interacting quantum field systems. The formula (5.1) is then simplified as

$$\begin{aligned} &\langle G|[TA_H^{\alpha}(t)\ldots B_H^{\gamma}(t')]_G|G\rangle \\ &\quad = \langle G_0|[T\hat{u}(\infty, -\infty)A^{\alpha}(t)\ldots B^{\gamma}(t')]_{G0}|G_0\rangle. \end{aligned} \tag{5.9}$$

It is worthwhile stressing that, although expression (5.9) is formally similar to the one for equilibrium, the operator $\hat{u}(\infty, -\infty)$ is different in the two cases. This is evident from (3.13) and (3.10). For a non-equilibrium system, $\hat{\mathcal{H}}_I$ is in general the sum of two different operators for $t > t_0$ and $t < t_0$. Besides, H_{1I} can have explicit time dependence through its coefficients. In the above procedure the initial-state correlations are interpreted as an effect of the evolution of the vacuum $|G_0\rangle$ in a fictitious past from $-\infty$ to t_0.

6. Concluding remarks

In this paper, we have developed a systematic perturbation theory in non-equilibrium TFD, by taking into account the initial-state correlations. Two methods were proposed; one is a double expansion method where time dependence of amplitudes and their

initial values are evaluated in different perturbation schemes while the other is a unified expansion method where the two perturbation schemes are combined by considering a fictitious past.

Since the initial condition in our formalism is determined by Ω which is in general not commutable with H, the time translation invariance is broken. This breakdown of the time translation invariance is represented in the perturbation scheme both in the fact that the time region of $t > t_0$ is considered and in the fact that the interaction Hamiltonian contains explicit time dependence. In particular, the self-energies are not functions of relative time. This fact leads to time-dependent energy shifts and particle distributions. A method to consider such effects has been discussed in [13] and [14] and a similar procedure can be applied in the present perturbation schemes. The practical applications of the present perturbation schemes will be discussed in future works.

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